

1 a  $\sin^2 x + \cos^2 x \equiv 1$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} + 1 \equiv \frac{1}{\cos^2 x}$$

$$\Rightarrow \tan^2 x + 1 \equiv \sec^2 x$$

b  $\sin^2 x + \cos^2 x \equiv 1$

$$\Rightarrow 1 + \frac{\cos^2 x}{\sin^2 x} \equiv \frac{1}{\sin^2 x}$$

$$\Rightarrow 1 + \cot^2 x \equiv \operatorname{cosec}^2 x$$

2 a  $\tan^2 A = \frac{1}{9}$

$$\sec^2 A = 1 + \frac{1}{9} = \frac{10}{9}$$

b  $\operatorname{cosec}^2 B = 1 + 2\sqrt{3} + 3$

$$= 4 + 2\sqrt{3}$$

$$\cot^2 B = (4 + 2\sqrt{3}) - 1$$

$$= 3 + 2\sqrt{3}$$

c  $\sec^2 C = \frac{9}{4}$

$$\tan^2 C = \frac{9}{4} - 1 = \frac{5}{4}$$

$$\tan C = \pm\sqrt{\frac{5}{4}} = \pm\frac{1}{2}\sqrt{5}$$

3 a  $3(1 + \tan^2 \theta) = 4 \tan^2 \theta$

$$\tan^2 \theta = 3$$

$$\tan \theta = \pm\sqrt{3}$$

$$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3} \text{ or } \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

b  $\sec^2 \theta - 1 - 2 \sec \theta + 1 = 0$

$$\sec^2 \theta - 2 \sec \theta = 0$$

$$\sec \theta (\sec \theta - 2) = 0$$

$$\sec \theta = 2 \text{ or } 0 \text{ [no solutions]}$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

c  $\operatorname{cosec}^2 \theta - 1 - 3 \operatorname{cosec} \theta + 3 = 0$

$$\operatorname{cosec}^2 \theta - 3 \operatorname{cosec} \theta + 2 = 0$$

$$(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta - 2) = 0$$

$$\operatorname{cosec} \theta = 1 \text{ or } 2$$

$$\sin \theta = \frac{1}{2} \text{ or } 1$$

$$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6} \text{ or } \frac{\pi}{2}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

d  $1 + \cot^2 \theta + \cot^2 \theta = 3$

$$\cot^2 \theta = 1$$

$$\cot \theta = \pm 1$$

$$\tan \theta = \pm 1$$

$$\theta = \frac{\pi}{4}, \pi + \frac{\pi}{4} \text{ or } \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

e  $1 + \tan^2 \theta + 2 \tan \theta = 0$

$$(\tan \theta + 1)^2 = 0$$

$$\tan \theta = -1$$

$$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

f  $1 + \cot^2 \theta - \sqrt{3} \cot \theta - 1 = 0$

$$\cot^2 \theta - \sqrt{3} \cot \theta = 0$$

$$\cot \theta (\cot \theta - \sqrt{3}) = 0$$

$$\cot \theta = 0 \text{ or } \sqrt{3}$$

$$\cos \theta = 0 \text{ or } \tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{2}, 2\pi - \frac{\pi}{2} \text{ or } \frac{\pi}{6}, \pi + \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$$

- 4 a**  $\sec^2 x - 1 - 2 \sec x - 2 = 0$   
 $\sec^2 x - 2 \sec x - 3 = 0$   
 $(\sec x + 1)(\sec x - 3) = 0$   
 $\sec x = -1$  or  $3$   
 $\cos x = -1$  or  $\frac{1}{3}$   
 $x = 180, -180$  or  $70.5, -70.5$   
 $x = -180^\circ, -70.5^\circ, 70.5^\circ, 180^\circ$
- b**  $2(1 + \cot^2 x) + 2 = 9 \cot x$   
 $2 \cot^2 x - 9 \cot x + 4 = 0$   
 $(2 \cot x - 1)(\cot x - 4) = 0$   
 $\cot x = \frac{1}{2}$  or  $4$   
 $\tan x = \frac{1}{4}$  or  $2$   
 $x = 14.0, 14.0 - 180$  or  $63.4, 63.4 - 180$   
 $x = -166.0^\circ, -116.6^\circ, 14.0^\circ, 63.4^\circ$
- c**  $\operatorname{cosec}^2 x + 5 \operatorname{cosec} x + 2(\operatorname{cosec}^2 x - 1) = 0$   
 $3 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x - 2 = 0$   
 $(3 \operatorname{cosec} x - 1)(\operatorname{cosec} x + 2) = 0$   
 $\operatorname{cosec} x = -2$  or  $\frac{1}{3}$  [no solutions]  
 $\sin x = -\frac{1}{2}$   
 $x = -30, 30 - 180$   
 $x = -150^\circ, -30^\circ$
- d**  $3 \tan^2 x - 3 \tan x + 1 + \tan^2 x = 2$   
 $4 \tan^2 x - 3 \tan x - 1 = 0$   
 $(4 \tan x + 1)(\tan x - 1) = 0$   
 $\tan x = -\frac{1}{4}$  or  $1$   
 $x = 180 - 14.0, -14.0$  or  $45, 45 - 180$   
 $x = -135^\circ, -14.0^\circ, 45^\circ, 166.0^\circ$
- e**  $\sec^2 x - 1 + 4 \sec x - 2 = 0$   
 $\sec^2 x + 4 \sec x - 3 = 0$   
 $\sec x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$   
 $\cos x = \frac{1}{-2 \pm \sqrt{7}}$   
 $\cos x = -0.2153$  or  $1.5486$  [no solutions]  
 $x = 180 - 77.6, 77.6 - 180$   
 $x = -102.4^\circ, 102.4^\circ$
- f**  $2 \cot^2 x + 3(1 + \cot^2 x) = 4 \cot x + 3$   
 $5 \cot^2 x - 4 \cot x = 0$   
 $\cot x(5 \cot x - 4) = 0$   
 $\cot x = 0$  or  $\frac{4}{5}$   
 $\cos x = 0$  or  $\tan x = \frac{5}{4}$   
 $x = 90, -90$  or  $51.3, 51.3 - 180$   
 $x = -128.7^\circ, -90^\circ, 51.3^\circ, 90^\circ$
- 5 a**  $\operatorname{cosec}^2 2x - 1 + \operatorname{cosec} 2x - 1 = 0$   
 $\operatorname{cosec}^2 2x + \operatorname{cosec} 2x - 2 = 0$   
 $(\operatorname{cosec} 2x + 2)(\operatorname{cosec} 2x - 1) = 0$   
 $\operatorname{cosec} 2x = -2$  or  $1$   
 $\sin 2x = -\frac{1}{2}$  or  $1$   
 $2x = 180 + 30, 360 - 30, 540 + 30,$   
 $720 - 30$  or  $90, 360 + 90$   
 $= 90, 210, 330, 450, 570, 690$   
 $x = 45^\circ, 105^\circ, 165^\circ, 225^\circ, 285^\circ, 345^\circ$
- b**  $8(1 - \cos^2 x) + \sec x = 8$   
 $8 \cos^2 x = \sec x$   
 $\cos^3 x = \frac{1}{8}$   
 $\cos x = \frac{1}{2}$   
 $x = 60, 360 - 60$   
 $x = 60^\circ, 300^\circ$
- c**  $\frac{3}{\sin^2 x} - 4 \sin^2 x = 1$   
 $4 \sin^4 x + \sin^2 x - 3 = 0$   
 $(4 \sin^2 x - 3)(\sin^2 x + 1) = 0$   
 $\sin^2 x = \frac{3}{4}$  or  $-1$  [no solutions]  
 $\sin x = \pm \frac{\sqrt{3}}{2}$   
 $x = 60, 180 - 60$  or  $180 + 60, 360 - 60$   
 $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$
- d**  $9(1 + \tan^2 x) - 8 = 1 + \cot^2 x$   
 $9 \tan^2 x = \cot^2 x$   
 $\tan^4 x = \frac{1}{9}$   
 $\tan^2 x = \frac{1}{3}$  or  $-\frac{1}{3}$  [no solutions]  
 $\tan x = \pm \frac{1}{\sqrt{3}}$   
 $x = 30, 180 + 30$  or  $180 - 30, 360 - 30$   
 $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

$$\begin{aligned}
 6 \quad \mathbf{a} \quad \text{LHS} &= 1 + \cot^2 x - (1 + \tan^2 x) \\
 &= \cot^2 x - \tan^2 x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \text{LHS} &= \cos^2 x - 4 + 4 \sec^2 x \\
 &= \cos^2 x - 4 + 4(1 + \tan^2 x) \\
 &= \cos^2 x + 4 \tan^2 x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad \text{LHS} &= \tan^2 x + 2 + \cot^2 x \\
 &= \sec^2 - 1 + 2 + \text{cosec}^2 x - 1 \\
 &= \sec^2 x + \text{cosec}^2 x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} \quad \text{LHS} &= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \\
 &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} \\
 &= \frac{1}{\cos^2 x \sin^2 x} \\
 &= \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} \\
 &= \sec^2 x \text{ cosec}^2 x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \text{LHS} &= \cot^2 x - 2 \cot x + 1 \\
 &= \text{cosec}^2 x - 2 \cot x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad \text{LHS} &= 1 + \tan^2 x - (1 - \cos^2 x) \\
 &= \tan^2 x + \cos^2 x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} \quad \text{LHS} &= \sin^2 x - 2 \sin x \sec x + \sec^2 x \\
 &= \sin^2 x - 2 \tan x + 1 + \tan^2 x \\
 &= \sin^2 x + (\tan x - 1)^2 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{h} \quad \text{LHS} &= \sec^2 x (1 + \tan^2 x) + \tan^2 x (\sec^2 x - 1) \\
 &= \sec^2 x + \sec^2 x \tan^2 x + \sec^2 x \tan^2 x - \tan^2 x \\
 &= 1 + \tan^2 x + 2 \sec^2 x \tan^2 x - \tan^2 x \\
 &= 2 \sec^2 x \tan^2 x + 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad 4 \sec^2 x - \sec x + 2 \tan^2 x = 0 &\Rightarrow 4 \sec^2 x - \sec x + 2(\sec^2 x - 1) = 0 \\
 &6 \sec^2 x - \sec x - 2 = 0 \\
 &(3 \sec x - 2)(2 \sec x + 1) = 0 \\
 &\sec x = -\frac{1}{2}, \frac{2}{3}
 \end{aligned}$$

for real values of  $x$ ,  $|\sec x| > 1 \therefore$  no real solutions

$$\begin{aligned}
 8 \quad \mathbf{a} \quad \text{LHS} &= \frac{1}{\sin x} \times \frac{1}{\cos x} - \frac{\cos x}{\sin x} \\
 &= \frac{1 - \cos^2 x}{\sin x \cos x} \\
 &= \frac{\sin^2 x}{\sin x \cos x} \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \text{cosec } x \sec x - \cot x &= 3 \\
 \tan x &= 3 \\
 x &= 71.6, 180 + 71.6 \\
 x &= 71.6^\circ, 251.6^\circ
 \end{aligned}$$